Higher Mathematics - Practice Examination D

Please note … the format of this practice examination is different from the current format. The paper timings are different and calculators can be used throughout.

MATHEMATICS

Higher Grade - Paper I

Time allowed - 2 hours

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. Calculators may be used.
3. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\,,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(\,a\,,\,b\,)$ and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned}
\sin\left(A \pm B\right) &= \sin A \cos B \pm \cos A \sin B \\
\cos\left(A \pm B\right) &= \cos A \cos B \mp \sin A \sin B \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A \\
\sin 2A &= 2\sin A \cos A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

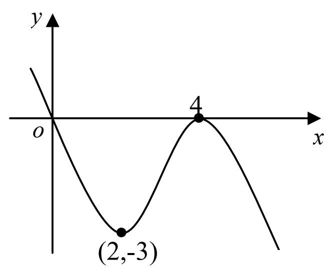
1. The parallelogram PQRS has three of its vertices as P(1,5) , Q(-1,-3) and R(3,-1).

(a) Establish the coordinates of the fourth vertex S. **(1)**

(b) Hence find the equation of the diagonal QS. **(2)**

2. Find $\frac{dy}{dx}$, given that $y = \frac{1}{8}(1+2x)^4 - \cos 3x$. **(4)**

3. Part of the graph of $y = f(x)$ is shown in the diagram.



Sketch the graph of $y = -f(x+2)$, showing all the relevant points.

(3)

4. The functions *f* and *g* are defined on suitable domains and are given as

$$f(a) = 3a - 2b \quad \text{and} \quad g(a) = 5(a + 2b) \text{ , where } b \text{ is a constant.}$$

Find an expression for $g\left(f(a)\right)$ in its simplest form. **(2)**

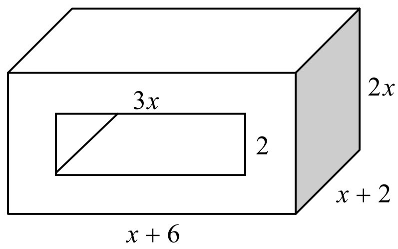
5. A recurrence relationship is defined as $U_{n+1} = 0 \cdot 5U_n + 16$ with $U_0 = 128$

(a) Find the limit (L) of this sequence. **(1)**

(b) Given that $U_n - L = 6$, find n . **(3)**

6. A small plastic block has a rectangular whole punched through it as shown.

All the lengths are in centimetres.



(a) Given that the volume of plastic in the block is 80 cubic centimetres, show, that to find x , the equation $x^3 + 5x^2 + 6x - 40 = 0$ must be solved. **(2)**

(b) Solve the equation and show that it has only one real root. Hence state the outside dimensions of the block. **(4)**

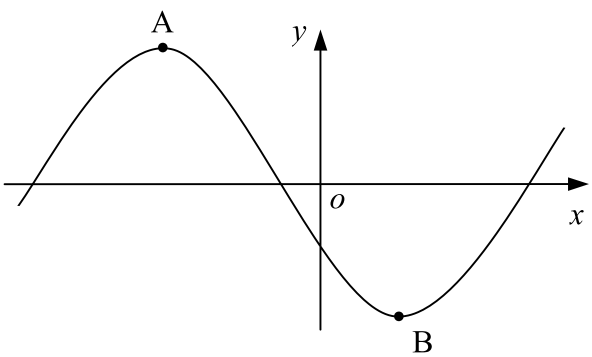
7. Find the equation of the tangent to the curve $y = x^2 + 3x + 5$ at the point where the curve crosses the y-axis. **(5)**

8. A logarithmic equation is given as

$$\tfrac{1}{2}\log_x(3x-2) = 1, \text{ where } x \neq 1.$$

Solve this equation for x. **(4)**

9. The diagram shows a sketch of the graph of $y = \frac{1}{3}x^3 + x^2 - 8x - \frac{26}{3}$.



(a) Find algebraically the coordinates of the two stationary points A and B. **(5)**

(b) Find the equation of the line AB and hence prove that this line passes through one of the points where the curve crosses the x-axis. **(4)**

10. A circle has as its equation $x^2 + y^2 + 6x - 4y = 0$.

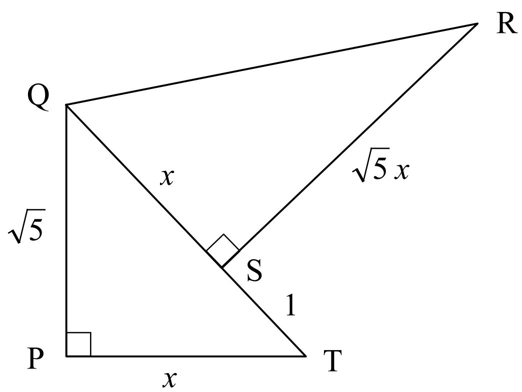
Find the equation of the tangent at the point P(-1,5) on the circle. **(4)**

11. The gradient of the tangent at any point (x , y) on a curve is given by $\frac{dy}{dx} = 2 + \frac{8}{x^2}$.

Given that the point (1 , 4) lies on this curve, express y in terms of x. **(4)**

12. The diagram features two right-angled triangles positioned as shown.

QS = PT = x , ST = 1 , PQ = $\sqrt{5}$ and SR = $\sqrt{5}\,x$.



(a) By applying Pythagoras' Theorem to ΔPQT , form an equation in x and solve it to find x . **(2)**

(b) Hence calculate the length of QR . **(1)**

(c) Show that the exact value of Sin PQR is given as

$$\text{Sin}\, PQR = \tfrac{7}{18}\sqrt{6} .$$ **(4)**

[END OF QUESTION PAPER]

1. (a) For finding S(5,7) (by stepping-out .... or equiv.) .... (1)

[1 mark]

(b) For gradient ... $m_{QS} = \frac{10}{6} = \frac{5}{3}$ .... (1)

For equ. $y - 7 = \frac{5}{3}(x - 5)$, (or equiv.), to $3y = 5x - 4$ .... (1)

[2 marks]

2. For .... $4 \times \frac{1}{8}(1+2x)^3 \times 2 = (1+2x)^3$ .... (2)

(*for* $4\times$ *and power of* 3 .. (1), *for* $\times 2$ .. (1))

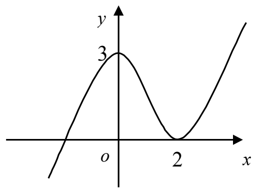
For $-(-\sin 3x \times 3) = 3\sin 3x$ .... (2)

(*for* $-\sin 3x \times 3$ ... (1) , *for* $-(-) = +$ ... (1)) **[4 marks]**

3. For knowing to reflect in x-axis .... (1)

For knowing to move 2 units to the left .... (1)

For completing the sketch and marking the two points .... (1)



[3 marks]

4. For $g\left(f(a)\right) = 5(3a - 2b) + 10b$ (or equivalent) .... (1)

For simplifying to answer .... $g\left(f(a)\right) = 15a$ .... (1)

[2 marks]

5. (a) For $L = \dfrac{b}{1-a}$ (or equiv.) $= \dfrac{16}{1-0\cdot 5} = 32$ .... (1)

[1 mark]

(b) For realising $U_n = 38$ (stated or implied) and starting with $U_1 = ..$ .... (1)

For $U_1 = 0{\cdot}5\,(128) + 16 = 80$ .... (1)

For successive lines of working until $U_4 = 38 \;\therefore\; n = 4$ .... (1)

[3 marks]

6. (a) For $2x(x+2)(x+6) - 6x(x+2) = 80$ (or eqiv.) .... (1)

For simplifying to ans. given ... $x^3 + 5x^2 + 6x - 40 = 0$ .... (1)

[2 marks]

(b) For deciding on method e.g. synthetic division .... (1)

For finding that 2 gives a remainder $= 0 \Rightarrow x = 2$. .... (1)

For using the quotient $x^2 - 7x + 20 = 0$ and showing that <u>it</u> has no real roots (i.e. $b^2 - 4ac < 0$) , $\therefore$ *no more roots* .... (1)

For final ans. $x = 2$ .... *block* $\Rightarrow 8 \times 4 \times 4\,cm$ .... (1)

[4 marks]

7. For obtaining y-intercept (0,5) .... (1)

For knowing to differentiate for gradient .... (1)

For diff. correctly to ... $\dfrac{dy}{dx} = 2x + 3 = m$ .... (1)

For sub. $x = 0$ into derivative to find that $m = 3$ .... (1)

For sub. $m = 3$ *and* (0,5) into equ. of line to ans..... $y = 3x + 5$ .... (1)

[5 marks]

8. For lifting to power .... $\log_x (3x-2)^{\frac{1}{2}} = 1$ .... (1)

For removing logs .... $x^1 = (3x-2)^{\frac{1}{2}}$ .... (1)

For squaring both sides .... $x^2 = 3x-2$ .... (1)

Solving to answer $((x-2)(x-1)=0 \quad \therefore \quad x=2$ .... (1)

[4 marks]

9. (a) For knowing to differentiate and solve to zero .... (1)

For differentiating correctly to .. $\frac{dy}{dx} = x^2+2x-8 = 0$ .... (1)

For solving to $x=-4$ *or* $x=2$ .... (1)

For completed points (1 mark each) A(-4,18) and B(2,-18) .... (2)

[5 marks]

(b) For gradient of AB .... $m=-6$ .... (1)

For equ. of AB .... $y-18=-6(x+4) \Rightarrow y=-6x-6$ or equiv. .... (1)

For finding where line crosses x-axis i.e (-1,0) .... (1)

For sub. $x = -1$ into equ. of curve to show that $y=0$, etc. .... (1)

(*i.e. proving that (-1,0) lies on curve and the line*) **[4 marks]**

10. For drawing out the centre C(-3,2) .... (1)

For finding the gradient of the radius $m_{CP} = \frac{3}{2}$ .... (1)

For gradient of tangent $m_{tan} = -\frac{2}{3}$ .... (1)

For ans. $y-5 = -\frac{2}{3}(x+1) \Rightarrow 2x+3y=13$ (or equiv.) .... (1)

[4 marks]

11. For knowing to integrate for "$y=$" .... (1)

For integrating to .... $y = 2x + \frac{8x^{-1}}{-1} + C$ (or equiv.) .... (1)

For knowing to sub. to find C .. i.e. $4 = 2(1) - \frac{8}{1} + C$ .... (1)

For $C = 10 \quad \therefore \quad y = 2x - \frac{8}{x} + 10$ ..... (or equiv.) .... (1)

[4 marks]

12. (a) For .... $(x+1)^2 = x^2 + (\sqrt{5})^2$ .... (1)

For answer $x=2$ .... (1)

[2 marks]

(b) For ..... $QR^2 = 2^2 + (2\sqrt{5})^2 \Rightarrow \therefore QR = \sqrt{24}$ *or* $2\sqrt{6}$ .... (1)

[1 mark]

(c) For knowing to use ....

$\sin PQR = \sin(a+b) = \sin a \cos b + \cos a \sin b$ (or equiv.) .... (1)

For lifting correct values to..$= \frac{2}{3} \cdot \frac{2}{2\sqrt{6}} + \frac{\sqrt{5}}{3} \cdot \frac{2\sqrt{5}}{2\sqrt{6}}$ (or equiv.) .... (1)

For simplifying to $\sin PQR = \frac{7}{3\sqrt{6}}$ (or equiv.) .... (1)

For rationalising denom. to final ans ...... Sin $PQR = \frac{7}{18}\sqrt{6}$ .... (1)

[4 marks]

Total : 55 marks

Higher Mathematics - Practice Examination D

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MATHEMATICS

Higher Grade - Paper II

Time allowed - 2 hours 40 mins

Read Carefully

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3. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad where \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad and \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned} \sin\left(A \pm B\right) &= \sin A \cos B \pm \cos A \sin B \\ \cos\left(A \pm B\right) &= \cos A \cos B \mp \sin A \sin B \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \\ \sin 2A &= 2 \sin A \cos A \end{aligned}$$

Table of standard derivatives:

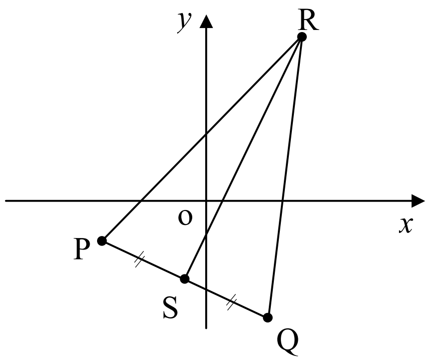
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

All questions should be attempted

1. Triangle PQR has as its vertices P(-6,-3) , Q(2,-7) and R(5,9) .



(a) Find the equation of the median RS. **(3)**

(b) Show that this median is at right-angles to side PQ. **(2)**

(c) What type of triangle is PQR? **(1)**

2. The functions f and g, defined on suitable domains, are given as

$$f(x) = \tfrac{1}{2}(2 + x^2) \quad \text{and} \quad g(x) = k(6x - 3).$$

(a) Given that $f(4) = g(5)$, find the value of k. **(2)**

(b) Find an expression for $g(f(x))$. **(2)**

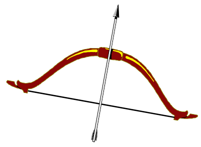
(c) Find x when $g(f(x)) = 2g(x)$. **(2)**

3. Solve algebraically the equation

$$3\sin x^\circ + 4\sin 2x^\circ = 0, \qquad 0 < x < 360.$$ **(6)**

4. Evaluate $\displaystyle\int_0^3 \frac{1}{x}(x^3 - 2x^2)\,dx$ **(4)**

5. The height, H metres, after t seconds, of an arrow fired vertically upwards from a toy bow is given by the equation



$$H = 40t - 16t^2 .$$

(a) Find the velocity of the arrow after 1 and after 2 seconds.

Explain the significance of the change of sign between these two velocities. **(4)**

(b) Find the maximum height to which the arrow rises. **(3)**

6. In a commercial laundry a certain material is washed in a solution of water and bleach. This washing solution is kept in a large vat and is continuously recycled through the cleaning machines.The solution is renewed each day.

At the beginning of each day fresh bleach and clean water are mixed in the vat. The initial strength of the bleach is 20 units/gallon.

It has been found, that in every hour of use, the bleach loses 12% of its strength.

(a) Calculate the strength of the bleach after 4 hours of use. **(3)**
Give your answer correct to two decimal places.

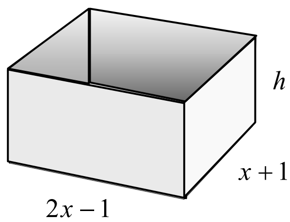
(b) It is known that if the strength of the bleach drops below 10 units/gallon the solution will no longer be an efficient cleaning agent.

The company decide to add a quantity of fresh bleach to the solution every 4 hours . This has the immediate effect of raising the overall strength of the bleach by 6 units/gallon.

Comment on whether or not this policy is effective. **(4)**
Your answer must be accompanied with the appropriate working.

7. A rectangular water tank has a volume of $22x - 27$ cubic metres.

Its base has dimensions $2x - 1$ metres by $x + 1$ metres as shown.



(a) From this information, and from the fact that $h = \dfrac{V}{lb}$,

write down a simple equation, in x ,for the height, h , of the tank. **(1)**

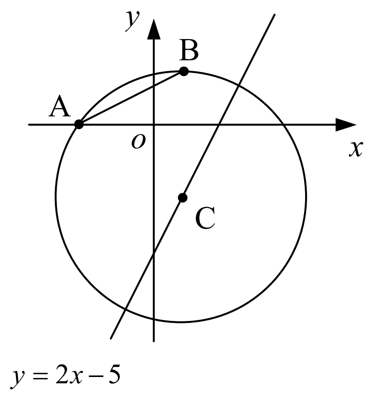
(b) Hence show that this equation can be written in the form

$$2hx^2 + (h - 22)x + (27 - h) = 0$$ **(3)**

(c) Show that if $h = 2$ this equation has equal roots. **(3)**

(d) Using this value for h , solve the equation for x and hence state the dimensions and volume of the tank. **(2)**

8. The points A(-3,0) and B(1,2) both lie on the circumference of a circle, centre C, as shown.

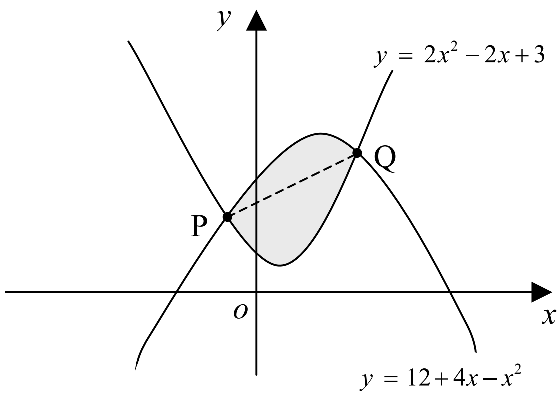


(a) Find the equation of the perpendicular bisector of the chord AB. **(3)**

(b) Hence find the coordinates of the centre of the circle, C, given that the line with equation $y = 2x - 5$ passes through C. **(3)**

(c) Find the equation of the circle. **(3)**

9. The diagram shows the parabolas $y = 12 + 4x - x^2$ and $y = 2x^2 - 2x + 3$.



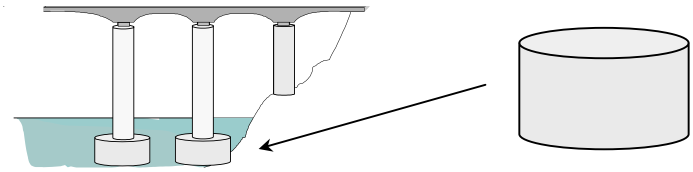
(a) Establish the coordinates of the two intersection points P and Q. **(3)**

(b) Find the area enclosed between the two curves. **(4)**

(c) Find the equation of the line PQ **(1)**

(d) Hence show that the line PQ splits the shaded area in the ratio 2 : 1 . **(4)**

10. One of the concrete supports for a small road bridge is cylindrical in shape.



Its total surface area, A_s , is 24 square metres.

(a) Given that the formula for the surface area, A_s , of a solid cylinder, is given as

$$A_s = 2\pi r^2 + 2\pi r h ,$$

show that, for this cylinder, $h = \dfrac{12}{\pi r} - r$, where h is its height and r is the radius of its base. **(2)**

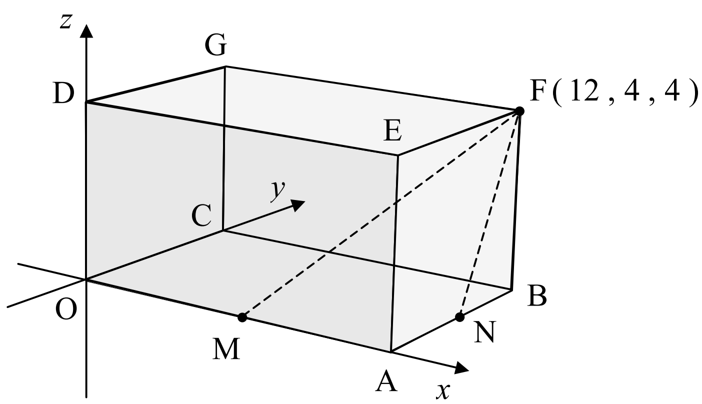
(b) Hence show that the volume of this cylinder can be expressed as

$$V(r) = 12r - \pi r^3 .$$ **(2)**

(c) Find the area of the base when the volume of the cylinder is at a maximum. **(5)**

11. A cuboid with dimensions 12cm by 4cm by 4cm is placed relative to a set of coordinate axes as shown in the diagram. F has coordinates (12 , 4 , 4)

M is the mid-point of OA and N is the mid-point of AB.



(a) Write down the coordinates of M and N . **(1)**

(b) Calculate the size of angle MFN . **(6)**

[END OF QUESTION PAPER]

1.	(a)	For coordinates of S .... S(-2,-5)	....	(1)
		For gradient of median ..... $m_{RS} = 2$	....	(1)
		For equ. of median ...... ans .... $y = 2x - 1$	....	(1)
				[3 marks]
	(b)	For selecting a strategy i.e. $m_1 \times m_2 = -1$	....	(1)
		For proving .... i.e. $2 \times -\frac{1}{2} = -1 \quad \therefore \quad right - angled$	....	(1)
				[2 marks]
	(c)	For ans .... isosceles	....	(1)
				[1 mark]
2.	(a)	For $f(4) = 9$ and $g(5) = 27k$ (or equiv.)	....	(1)
		For equating and solving to answer .... $k = \frac{1}{3}$	....	(1)
				[2 marks]
	(b)	For $g(\frac{1}{2}(2 + x^2)) = 2\left[\frac{1}{2}(2 + x^2)\right] - 1$ (or equiv.)	....	(1)
		For simplifying to answer $g(f(x)) = x^2 + 1$	....	(1)
				[2 marks]
	(c)	For .... $x^2 + 1 = 2(2x - 1)$ (or equiv.)	....	(1)
		For solving to answer ...... i.e. $x^2 - 4x + 3 = 0 \Rightarrow (x-1)(x-3) = 0 \quad \therefore \quad x = 1 \quad or \quad x = 3$	....	(1)
				[2 marks]
3.	For	$3\sin x + 4(2\sin x\cos x) = 0$	....	(1)
	For	$3\sin x + 8\sin x\cos x = \sin x(3 + 8\cos x) = 0$	....	(1)
	For	$\sin x = 0$ or $\cos x = -\frac{3}{8}$	....	(1)
	Then for	$\sin x = 0 \quad then \quad x = 180^\circ$	....	(1)
	Then for	$\cos x = -\frac{3}{8} \quad then \quad x = 112\cdot0^\circ \quad or \quad 248\cdot0^\circ$ (1 each)	....	(2)
				[6 marks]
4.	For	$\int_0^3 (x^2 - 2x)\, dx$	....	(1)
	then	$\left[\frac{x^3}{3} - \frac{2x^2}{2}\right]_0^3$ (or equivalent)	....	(1)
	then	$\left(\frac{27}{3} - \frac{18}{2}\right) - (0)$	....	(1)
	For answer	0	....	(1)
				[4 marks]

5. a) For knowing to differentiate for velocity .... (1)

For substituting 1 and 3 into derivative ... $v'(t) = 40 - 32t$ .... (1)

For answers 8 m/s and -24 m/s .... (1)

For explanation i.e rising then falling, different direction, etc. .... (1)

[4 marks]

(b) For knowing to solve derivative to zero .... (1)

For $\therefore\ t = \frac{5}{4}$ seconds @ maximum .... (1)

For sub. in $H = ...$ to answer $H = 25$ metres .... (1)

[3 marks]

<u>**or**</u>

For solving $40t - 16t^2 = 0$ .... *(1)*

For t half-way between roots 0 and $\frac{5}{2}$, $\therefore\ t = \frac{5}{4}s$ .... *(1)*

For sub. in H = ... to answer H = 25 metres .... *(1)*

6. (a) For correct multiplier i.e. $0 \cdot 88$ .... (1)

For $\quad Strength = (0 \cdot 88)^4 \times 20$ .... (1)

$= 11 \cdot 99$ *units* .... (1)

(*pupils may use four lines of working*) **[3 marks]**

(b) For setting up recurr. .. $U_{n+1} = (0 \cdot 88)^4 . U_n + 6$ (or equiv.) .... (1)

For setting out at least four lines of calculations .... (1)

For knowing to look at **lower value** before adding 6 .... (1)

For discovering ans.... that by end of the fourth cycle (i.e. 16 hours) strength is $\approx 9 \cdot 64$ (immediately before 6 is added)

...... policy is <u>not</u> acceptable .... (1)

(for only looking at upper value and .... ans. o.k., fundamental error, ²⁄₄) **[4 marks]**

** note : limit cannot be applied as after the 6 cycles ($\times$ 4 hrs) situation is re-set.

7. (a) For ans. $h = \dfrac{V}{lb} = \dfrac{22x - 27}{(2x-1)(x+1)}$ (or equiv.) .... (1)

[1 mark]

(b) For knowing to change towards standard quad. form .... (1)

For ..... $2hx^2 + hx - h = 22x - 1$ .... (1)

For arranging to ans ...... $2hx^2 + (h - 22)x + (27 - h) = 0$ .... (1)

[3 marks]

(c) For knowing to sub. $h = 2$ into equation .... (1)

For stating or showing its a perfect square $(2x - 5)^2 = 0$ .... (1)

and explaining it has only one root .... (1)

(*pupils may use discriminant ... for equal roots* $b^2 - 4ac = 0$ .... (1) **[3 marks]**

... for showing $400 - 400 = 0$ ... (1)

(d) For solving ...... $(2x - 5)^2 = 0$ ... ans $x = 2 \cdot 5$ metres .... (1)

For ..... $\therefore$ dimensions $4 \times 3 \cdot 5 \times 2$ *metres* and $V = 28\ m^3$ .... (1)

[2 marks]

8. (a) For $m_{AB} = \frac{1}{2}$ then $m_{perp} = -2$ .... (1)
For mid-point ...... M(-1,1) .... (1)
For correct gradient and point to .. $y - 1 = -2(x+1) \Rightarrow 2x + y + 1 = 0$ ...... (1)
[3 marks]

(b) For knowing perp. bisec. passes through centre (stated or implied) .... (1)
For knowing to solve as a system $\left\{\begin{array}{l} 2x + y = -1 \\ y = 2x - 5 \end{array}\right\}$ .... (1)
For solving to answer ...... C(1,-3) .... (1)
[3 marks]

(c) For finding r^2 by Pyth. <u>or</u> noticing that B is vert. above C , r =5 .... (1)
For knowing to sub. r^2 and C(1,-3) into $(x-a)^2 + (y-b)^2 = r^2$ ..... (1)
For final answer .... $(x-1)^2 + (y+3)^2 = 25$ .... (1)
[3 marks]

9. (a) For setting up the system .. $12 + 4x - x^2 = 2x^2 - 2x + 3$ (or equiv.) .... (1)
For simplifying to .. $3(x^2 - 2x - 3) = 0$ .... (1)
For solving to $x = -1$ *and* $x = 3$ then to P(-1,7) and Q(3,15) .... (1)
[3 marks]

(b) For setting-up corredt integral i.e. $\int_{-1}^{3} (-3x^2 + 6x + 9)\ dx$ .... (1)
For integrating to ..... A = $\left[-x^3 + 3x^2 + 9x\right]_{-1}^{3}$ .... (1)
For substituting in numbers .... (1)
For correct answer Area = 32 square units. .... (1)
[4 marks]

(c) For $m = 2$
then .... $y - 7 = 2(x+1) \Rightarrow y = 2x + 9$ (or equiv.) .... (1)
[1 mark]

(d) For correct int. i.e $\int_{-1}^{3} (2x+9) - (2x^2 - 2x + 3)\ dx$ (or equiv.) .... (1)
For simplifying and integrating to A $= \left[\dfrac{-2x^3}{3} + 2x^2 + 6x\right]_{-1}^{3}$ .... (1)
For substituting to answer ..... A = $21\frac{1}{3}$ units squared .... (1)
For proving 2 : 1 by $32 \div 3 = 10\frac{2}{3}$ .... $\times 2 = 21\frac{1}{3}$ (or equiv.) .... (1)
(*pupils may use line and other curve , giving* $A = 10\frac{2}{3}$ *, etc.*) **[4 marks]**

10. (a) For replacing A_s with 24 to ... $24 = 2\pi r^2 + 2\pi r h$ .... (1)

For making h the subject to answer .... (1)

[2 marks]

(b) For writing $V = \pi r^2 h = \pi r^2 \left[\dfrac{12}{\pi r} - r \right]$ .... (1)

For simplifying to given answer .... (1)

[2 marks]

(c) For knowing to differentiate .... (1)

For diff. correctly <u>and</u> knowing to solve to zero... $12 - 3\pi r^2 = 0$ .... (2)

For answer .... for max volume .. $r^2 = \dfrac{4}{\pi}$ or $r = \sqrt{\dfrac{4}{\pi}}$ .... (1)

For sub. in $A_{base} = \pi r^2$ .... $= \pi . \dfrac{4}{\pi} = 4$ sq. metres. .... (1)

[5 marks]

11. (a) For .... M(6,0,0) and N(12,2,0) .... (1)

[1 mark]

(b) For selecting correct vectors i.e. $\vec{FM}$ *and* $\vec{FN}$ .... (1)

For $\vec{FM} = \begin{pmatrix} -6 \\ -4 \\ -4 \end{pmatrix}$ *and* $\vec{FN} = \begin{pmatrix} 0 \\ -2 \\ -4 \end{pmatrix}$ .... (1)

For scalar product $\vec{FM} . \vec{FN} = 24$ .... (1)

For both magnitudes $\sqrt{68}$ *and* $\sqrt{20}$ (or equiv.) .... (1)

For $\cos\theta = \dfrac{24}{\sqrt{68} \times \sqrt{20}}$ (or equiv.) .... (1)

For ans. angle MFN $= 49 \cdot 4^\circ$ .... (1)

[6 marks]

Total : 82 marks