Education Resources

Circle

Higher Mathematics Supplementary Resources

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I can use the distance formula to find the distance between two points.

1. Find the distance between each of the pairs of points below. Leave your answer as a surd where appropriate.

(a) $A(3,4)$ $B(6,8)$ **(b)** $C(-2,0)$ $D(3,12)$ **(c)** $E(3,-1)$ $F(0,-5)$

(d) $G(0,4)$ $H(-3,-7)$ **(e)** $J(-3,9)$ $K(3,-1)$ **(f)** $P(2,-5)$ $Q(-1,7)$

2. A circle has diameter AB where $A(6,-2)$ $B(-3,5)$. Find the size of the radius of this circle.

3. The centre of three concentric circles is $(-5,3)$. Find the radius of each circle if:

(a) The smallest circle goes through the point $(-3,2)$.

(b) The middle circle goes through the point $(-7,5)$.

(c) The largest circle goes through the point $(8,7)$.

4. Two circles of the same size have centres $(-3,4)$ and $(2,-7)$ and touch at a single point.

Find the size of the radii of the circles.

R2 I can determine the equation of a circle given its centre and radius using $(x-a)^2 + (y-b)^2 = r^2$

1. Find the equation of the circles with:

(a) Centre $(3, 4)$ and radius 5
(b) Centre $(0, 0)$ and radius 8
(c) Centre $(0, 2)$ and radius 7
(d) Centre $(3, -8)$ and radius 6
(e) Centre $(-1, -5)$ and radius 10
(f) Centre $(-2, 0)$ and radius 3

2. Find the equation of the circles with:

(a) Centre $(0, 0)$ and diameter 3
(b) Centre $(-1, 4)$ and diameter 6
(c) Centre $(0, -5)$ and diameter 12
(d) Centre $(9, -7)$ and diameter 2
(e) Centre $(-3, -7)$ and diameter 5
(f) Centre $(4, 0)$ and diameter 7

R3 I can determine the centre and radius of a circle given its equation using $x^2 + y^2 + 2gx + 2fy + c = 0$.

State the centre and radius of each of these circles.

1. $x^2 + y^2 + 4x + 8y - 5 = 0$
2. $x^2 + y^2 - 2x + 10y + 3 = 0$
3. $x^2 + y^2 - 10x - 4y - 9 = 0$
4. $x^2 + y^2 - 25 = 0$
5. $x^2 + y^2 + x - 6y + 8 = 0$
6. $x^2 + y^2 + 3x - y - 6 = 0$

R4 I can determine if a point lies inside, outside or on the circle.

In each example below, the equation of a circle and a point are given. In each case, state whether the point does or does not lie on the circumference of the given circle.

1. $x^2 + y^2 + 4x - 6y - 16 = 0$ and $(0, -2)$.

2. $x^2 + y^2 - 2x + 10y + 3 = 0$ and $(3, 4)$.

3. $x^2 + y^2 - 10x - 4y - 9 = 0$ and $(-2, -5)$.

4. $x^2 + y^2 - 2x + 4y - 15 = 0$ and $(-1, 2)$.

5. $x^2 + y^2 - 10x - 12y + 61 = 0$ and $(-5, -6)$.

6. $x^2 + y^2 + 10x - 9y - 18 = 0$ and $(2, 5)$.

R5 I can use $g^2 + f^2 - c$ to determine whether an equation in the form $x^2 + y^2 + 2gx + 2fy + c = 0$ is an equation of a circle or not.

Which of the equation below represent the equation of a circle?

1. $x^2 + y^2 + 4x - 6y - 3 = 0$

2. $x^2 + y^2 - 8x + 6y = 0$

3. $x^2 + y^2 - 4x - 6y + 16 = 0$

4. $x^2 + y^2 + 2x - 8y + 24 = 0$

5. $x^2 + y^2 + 14x + 6y + 54 = 0$

6. $x^2 + y^2 - 6x + 2y + 15 = 0$

7. $x^2 + y^2 - 2x - 14y + 14 = 0$

8. $x^2 + y^2 - 4x + 10y + 37 = 0$

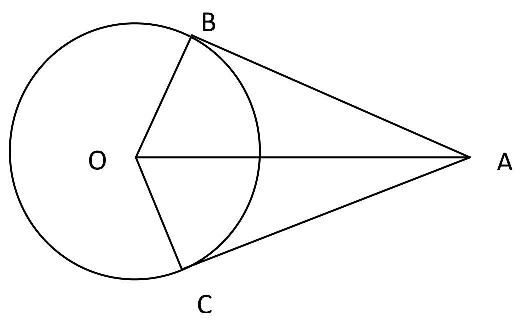
Section B

This section is designed to provide examples which develop Course Assessment level skills

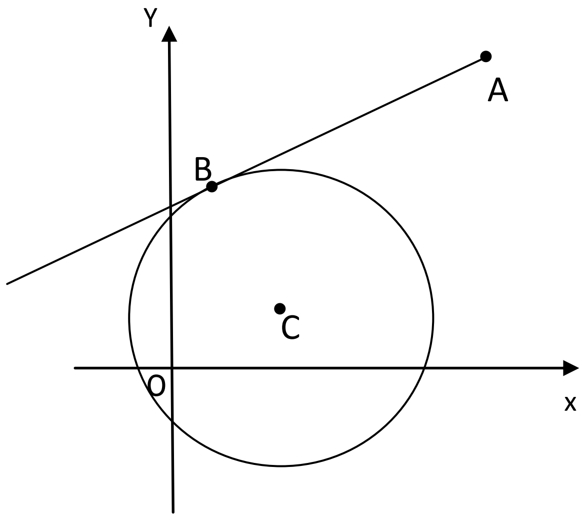
NR1 I can use circle equations in mixed problems.

1. The points A(6,3,1), B(8,4,-9) and C(3,1,k) lie on the circumference of a semicircle with AB as diameter. Find all the possible values of k.

2. In the diagram AB and AC are tangents from the point A(9,0) to the circle $x^2 + y^2 = 16$, with centre O. Find the area of the kite ABOC.



3. AB is a tangent at B to the circle with centre C and equation $(x-2)^2 + (y-2)^2 = 25$. The point A has coordinates (10,8). Find the area of triangle ABC.



4. A sports club awards trophies in the form of paperweights bearing the club crest. Diagram 1 shows the front view of one of these paperweights. Each is made from two different types of glass. The two circles are concentric and the base line is a tangent to the inner circle.

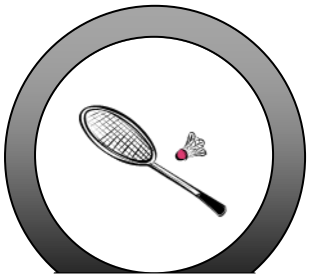


Diagram 1

(a) Relative to x, y coordinate axes, the equation of the outer circle is $x^2 + y^2 - 8x + 2y - 19 = 0$ and the equation of the base line is $y = -6$.
Show that the equation of the inner circle is
$x^2 + y^2 - 8x + 2y - 8 = 0$.

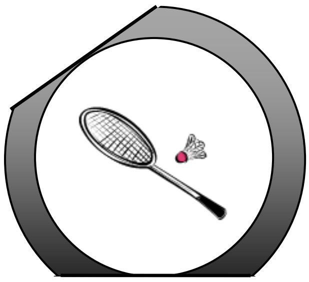


Diagram 2

(b) An alternative form of the paperweight is made by cutting off a piece of glass from the original design along a second line with equation $3x - 4y + 9 = 0$ as shown above in diagram 2.
Show that this line is a tangent to the inner circle and state the coordinates of the point of contact.

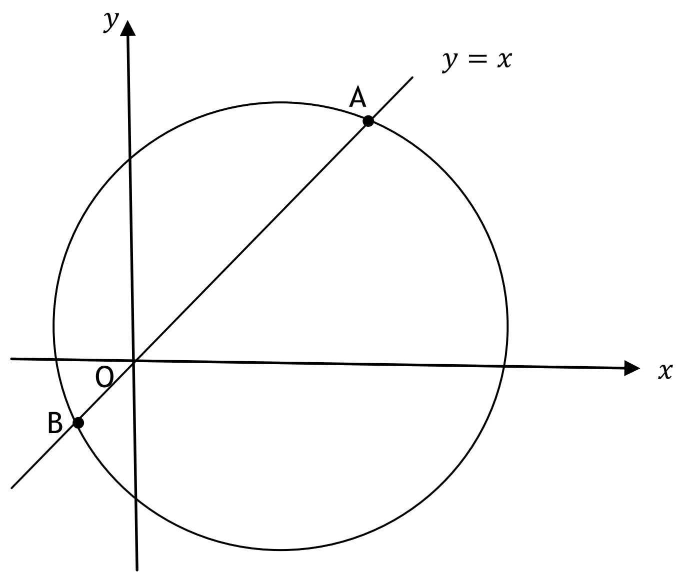
NR2 I can determine the point of intersection of a line and a circle or two circles.

1. A circle C_1 has centre A(1, 3) and radius $\sqrt{5}$. A circle C_2 has centre B(9, 7) and radius $3\sqrt{5}$.

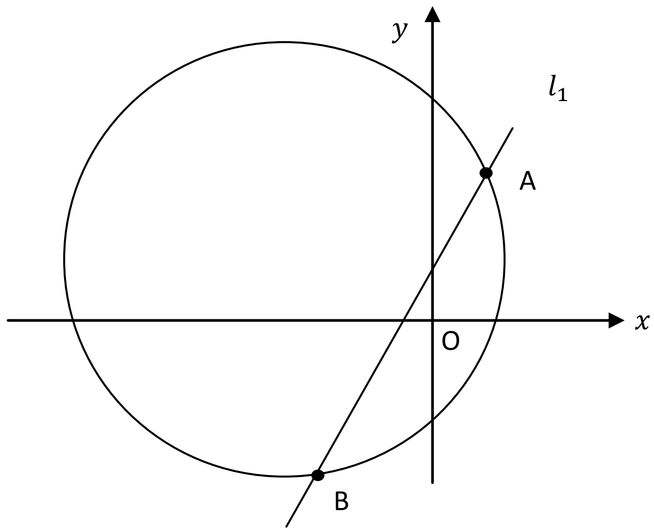
(a) Verify that C_1 touches C_2.
(b) Find the coordinates of X, the point of contact of C_1 and C_2 .
(c) Find the equation of the common tangent to C_1 and C_2 drawn through X.

2. The straight line $y = x$ cuts the circle $x^2 + y^2 - 6x - 2y - 24 = 0$ at A and B.

(a) Find the coordinates of A and B.
(b) Find the equation of the circle which has AB as diameter.

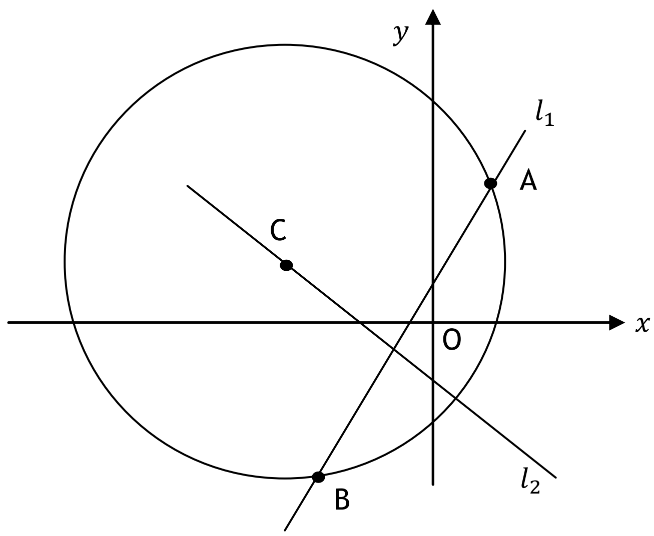


3. Diagram 1 shows a circle with equation $x^2 + y^2 + 10x - 2y - 14 = 0$ and a straight line, l_1, with equation $y = 2x + 1$.
The line intersects the circle at A and B.



(a) Find the coordinates of the points A and B.

(b) Diagram 2 shows a second line, l_2, which passes through the centre of the circle, C, and is at right angles to the line l_1.

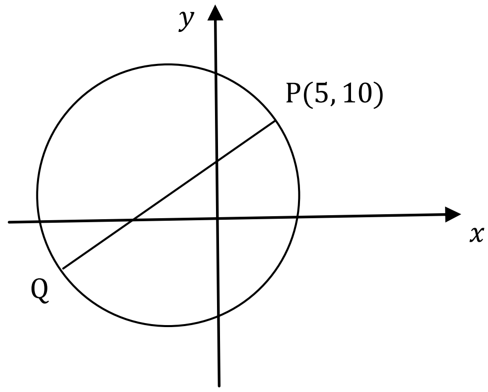


(i) Write down the coordinates of C.

(ii) Find the equation of the line l_2.

4. **(a)** Show that the point P(5,10) lies on circle C_1 with equation $(x+1)^2+(y-2)^2=100$.

(b) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q.



(c) Two circles, C_2 and C_3 touch C_1 at Q.

The radius of each of these circles is twice the radius of C_1.
Find the equations of circles C_2 and C_3.

5. Circle C_1 has equation $(x+1)^2+(y-1)^2=121$.
A circle C_2 with equation $x^2+y^2-4x+6y+p=0$ is drawn inside C_1.
The circles have no points of contact.
What is the range of values of p?

6. **(a)** Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

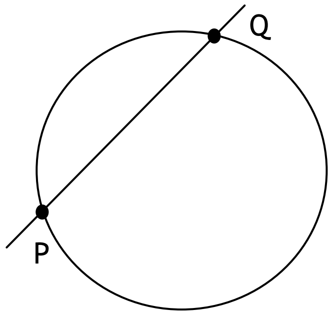


Diagram 1

Find the coordinates of P and Q.

(b) Diagram 2 shows the circle from **(a)** and a second congruent circle, which also passes through P and Q.

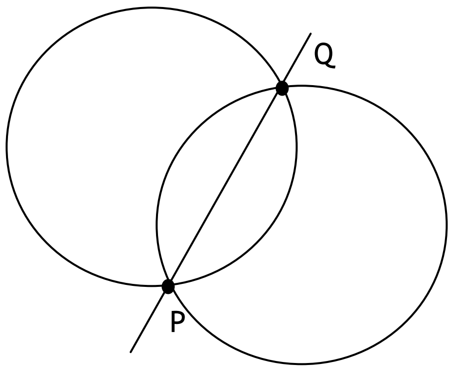


Diagram 2

Determine the equation of this second circle.

NR3	I can determine the point of intersection of a tangent to a circle using the criteria for tangency.

1. Find the possible values of k for which the line $x - y = k$ is a tangent to the circle $x^2 + y^2 = 18$.

2. Show that the line $x + y = 10$ is a tangent to the circle $x^2 + y^2 - 2x - 10y + 18 = 0$ and find the coordinates of the point of contact.

3. **(a)** Show that the equation of the circle which passes through (0, 0), (4, 0) and (0, -2) is $x^2 + y^2 - 4x + 2y = 0$.

(b) Show that the line with equation $y = 2x - 10$ is a tangent to this circle and state the coordinates of the point of contact.

4. A circle, centre C, has equation $x^2 + y^2 - 4x + 6y - 12 = 0$.

(a) Find the equation of the tangent at the point A(5, 1) on this circle.

(b) Show that the line through the point P(1, 4) at right angles to the tangent has equation $3y - 4x = 8$ and show that this line is also a tangent to the circle.

5. A, B and C are the points (-1, 1), (1, 2) and (4, 1) respectively. AP is a diameter of a circle, centre B.

(a) State the equation of the circle.

(b) Prove that CP is a tangent to the circle.

(c) D is the point (0, -1). Prove that CD is the other tangent to the circle from C.

NR4 I can determine the equation of a tangent to a circle.

1. Find the equation of the tangent to the circle $x^2 + y^2 - 3x + y - 16 = 0$ at the point (4,3).

2. **(a)** Find the equation of the circle, centre (9,-1), which passes through the point A (3,8).

(b) Obtain the equation of the tangent to this circle at A.

(c) Prove that this tangent passes through the centre of the circle with equation $x^2 + y^2 + 6x - 8y + 12 = 0$.

3. Prove that the line with equation $5x + y - 10 = 0$ is a tangent to the circle with equation $x^2 + y^2 - 16x + 8y + 54 = 0$ and find the coordinates of the point of contact.

4. **(a)** Find the coordinates of the centre, C, and the radius of the circle whose equation is $x^2 + y^2 - 2x - 4y - 3 = 0$.

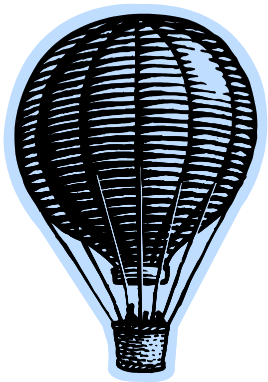
(b) If a tangent to the circle at the point A(3,4) is drawn:

(i) Find the equation of the tangent at A.

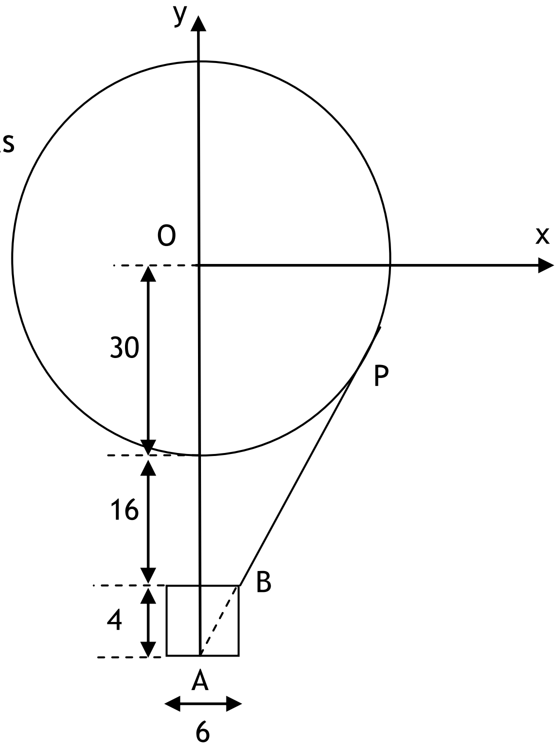
(ii) Prove that the point P(7,0) lies on the tangent.

(iii) Find the equation of the circle which passes through points C, A and P.

5.



A spherical hot-air balloon has a radius 30 feet. Cables join the balloon to the gondola which is cylindrical with diameter 6 feet and height 4 feet. The top of the gondola is 16 feet below the bottom of the balloon.



Coordinate axes are chosen as shown in the diagram.

One of the cables is represented by PB and PBA is a straight line.

(a) Find the equation of the cable PB.

(b) State the equation of the circle representing the balloon.

(c) Prove that this cable is a tangent to the balloon and find coordinates of P

NR5 I have experience of cross topic exam standard questions.

1. A and B are the points $(2, 2)$ and $(4, 8)$ respectively.

(a) Find the equation of the perpendicular bisector of AB.

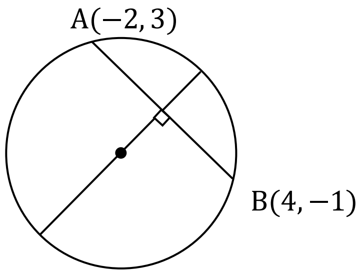
(b) Given that C, a point in the first quadrant equidistant from both axes, is the centre of the circle passing through A and B, find

(i) the coordinates of C;

(ii) the equation of the circle.

(c) Prove that the line $7x - y - 2 = 0$ is a tangent to this circle and state the coordinates of the point of contact.

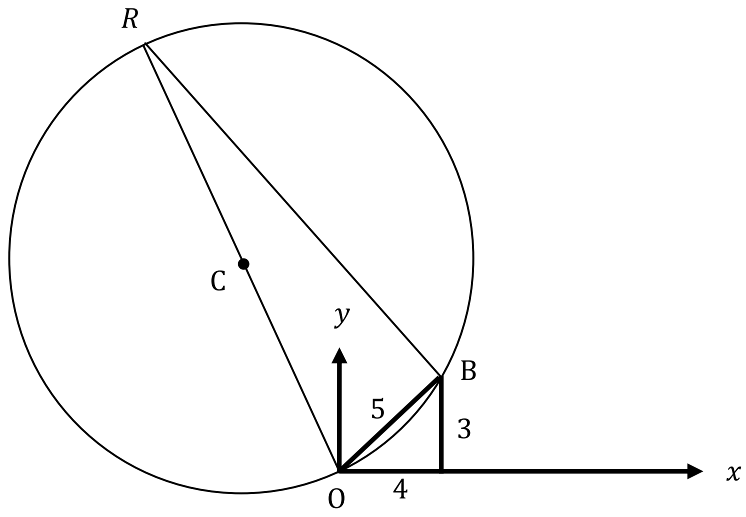
2. A circle passes through A$(-2, 3)$ and B$(4, -1)$.



Find the equation of the diameter which is perpendicular to the chord AB.

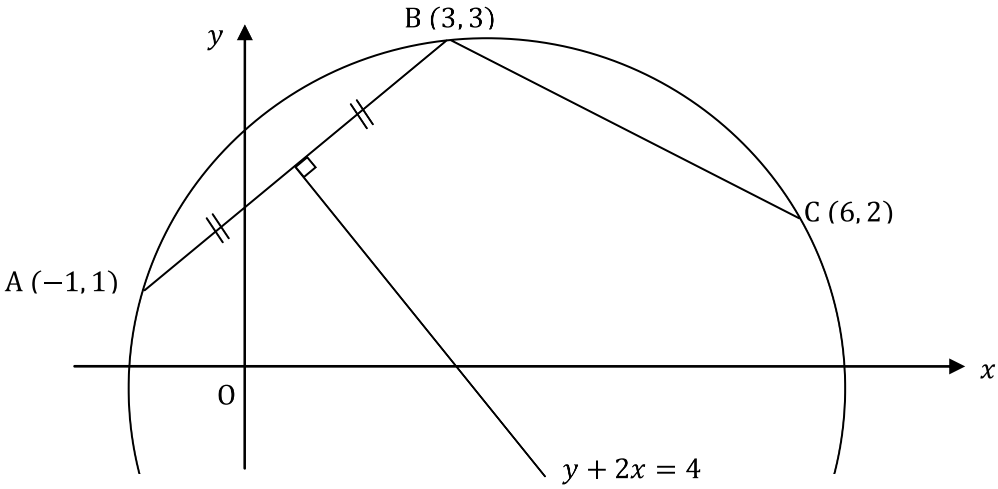
3. The right-angled triangle OAB with sides of length 3 cm, 4 cm and 5 cm is placed with one vertex at the origin O as shown in the diagram.

A circle of centre C with diameter RO of length 13 cm is drawn and passes through O and B.



What is the gradient of the line RO?

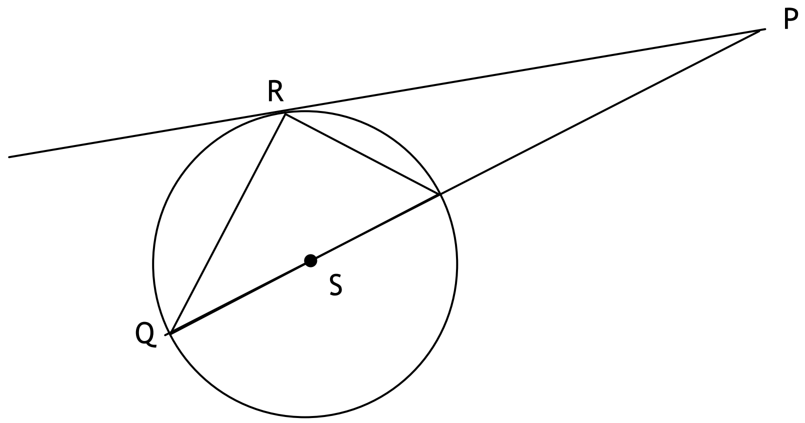
4. A is the point $(-1, 1)$, B is $(3, 3)$ and C is $(6, 2)$. The perpendicular bisector of AB has equation $y + 2x = 4$.



(a) Find the equation of the perpendicular bisector of BC.

(b) Find the centre and equation of the circle which passes through A, B and C.

5. P is a point on the tangent at R to a circle centre S. PS extends to cut the circle at Q.



If angle $\mathrm{RPQ} = 2\theta°$ and $\mathrm{PQ} = d$ units, prove that:

(a) angle $\mathrm{SQR} = (45 - \theta)°$

(b) by applying the Sine Rule to triangle PRQ,

$$\mathrm{RP} = \frac{d(\cos\theta° - \sin\theta°)}{\cos\theta° + \sin\theta°}$$

Answers

R1

1. **(a)** 5 **(b)** 13 **(c)** 5

(d) $\sqrt{130}$ **(e)** $2\sqrt{34}$ **(f)** $\sqrt{153}$

2. $\frac{\sqrt{130}}{2}$

3. **(a)** $\sqrt{5}$ **(b)** $2\sqrt{2}$ **(c)** $\sqrt{185}$

4. $\frac{\sqrt{146}}{2}$

R2

1. **(a)** $(x-3)^2+(y-4)^2=25$ **(b)** $x^2+y^2=64$

(c) $x^2+(y-2)^2=49$ **(d)** $(x-3)^2+(y+8)^2=36$

(e) $(x+1)^2+(y+5)^2=100$ **(f)** $(x+2)^2+y^2=9$

2. **(a)** $\mathbf{x^2+y^2=\frac{9}{4}}$ **(b)** $\mathbf{(x+1)^2+(y-4)^2=9}$

(c) $x^2+(y+5)^2=36$ **(d)** $(x-9)^2+(y+7)^2=4$

(e) $(x+3)^2+(y+7)^2=\frac{25}{4}$ **(f)** $(x-4)^2+y^2=\frac{49}{4}$

R3

1. $(-2,-4)\ r=5$ **2.** $(1,-5)\ r=\sqrt{23}$

3. $(5,2)\ r=\sqrt{38}$ **4.** $(0,0)\ r=5$

5. $\left(-\frac{1}{2},3\right)\ r=\sqrt{\frac{5}{4}}$ **6.** $\left(-\frac{3}{2},\frac{1}{2}\right)\ r=\sqrt{\frac{17}{2}}$

R4

1. *does* **2.** *does not*

3. *does not* **4.** *does*

5. *does not* **6.** *does not*

R5

1, 2, 5 and 7

NR1

1. k=-2 or k=-6
2. Area = 32.25 ($4\sqrt{65}$) units²
3. Area $= \frac{25\sqrt{3}}{2}$ units²
4. **(a)** proof **(b)** (1,3) - numbers involved are challenging!!

NR2

1. **(a)** $|AB| = 4\sqrt{5}$. Circles touch **(b)** $\mathrm{X}(3, 4)$

   (c) $x + 2y - 10 = 0$
2. **(a)** $\mathrm{A}(6, 6)\ \mathrm{B}(-2, -2)$ **(b)** $(x-2)^2 + (y-2)^2 = 32$
3. **(a)** $\mathrm{A}(1, 3)\ \mathrm{B}(-3, -5)$ **(b)(i)** $\mathrm{C}(-5, 1)$ **(ii)** $2y + x = -3$
4. **(a)** Lies on circle. **(b)** $3x + 4y = -45$ **(c)** $(x-5)^2 + (y-10)^2 = 400$ and $(x+19)^2 + (y+22)^2 = 400$
5. $-23 < p < 13$
6. **(a)** $\mathrm{P}(-3,\ -1)\ \mathrm{Q}(1,\ 7)$ **(b)** $(x+5)^2 + (y-5)^2 = 40$

NR3

1. k=6 or k=-6
2. proof; (3,7)
3. **(a)** proof **(b)** proof; (4,-2)
4. **(a)** $3x + 4y = 19$ **(b)** proof; (-2,0)
5. **(a)** $(x-1)^2 + (y-2)^2 = 5$ **(b)** $2x + y = 9$; proof **(c)** $x - 2y = 2$; proof

NR4

1. $5x + 7y = 41$

2. **(a)** $(x-9)^2 + (y+1)^2 = 117$ **(b)** $2x - 3y + 18 = 0$

(c) proof

3. (3, -5)

4. **(a)** centre (1,2) , r=$2\sqrt{2}$ **(b)** **(i)** $x + y = 7$ **(ii)** proof
(iii) $(x-4)^2 + (y-1)^2 = 10$

5. **(a)** $4x - 3y = 150$ **(b)** $x^2 + y^2 = 900$ **(c)** P (24, -18)

NR5

1. **(a)** $3y + x = 18$ **(b)(i)** $\left(\frac{9}{2}, \frac{9}{2}\right)$ **(ii)** $\left(x - \frac{9}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = \frac{25}{2}$

(c) Proof for tangency at $(1, 5)$

2. $2y - 3x = -1$

3. $\frac{-65}{16}$

4. **(a)** $2y - 6x = -22$

(b) Centre $(3, -2)$, Equation $(x-3)^2 + (y+2)^2 = 25$

5. **(a)** Proof **(b)** Proof